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**i860™ Microprocessor
Math Library
Reference Manual**

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Preface

The Intel i860™ Microprocessor delivers supercomputing performance in a single VLSI component. The 64-bit design of the i860 Microprocessor balances integer, floating point, and graphics performance for applications such as engineering workstations, scientific computing, 3-D graphics workstations, and multiuser systems. Its parallel architecture achieves high throughput with RISC design techniques, pipelined processing units, wide data paths, large on-chip caches, million-transistor design, and fast one-micron CHMOS IV silicon technology.

The Math Library is a package of procedures (called “primitives”) designed for use when writing programs for the i860 Microprocessor that use high-level mathematical functions.

This manual is a complete source of information about the use of the Math Library. It is useful both to software managers and software engineers. Software managers will learn how this product can accelerate development of applications for the i860 Microprocessor. Software engineers will find detailed descriptions of the use of the product.

How to Use This Manual

- Chapter 1, “Introduction” presents an overview of the Math Library and defines the notation of Chapter 2. Read this chapter for information common to all primitives.
- Chapter 2, “Math Primitives” specifies each of the primitives.

Related Documentation

The following literature contains additional information concerning the i860 Microprocessor:

- *i860 64-Bit Microprocessor* (Data Sheet), order number 240296
- *i860 64-Bit Microprocessor Programmer's Reference Manual*, order number 240329
- *i860 64-Bit Microprocessor Simulator and Debugger Reference Manual*, order number 240437
- *i860 64-Bit Microprocessor Assembler and Linker Reference Manual*, order number 240436

Notation and Conventions

| | |
|----------------------|--|
| <i>italic</i> | In mathematical notation, scalar variables are printed in roman italic type; for example, <i>scalar</i> . |
| roman bold | In mathematical notation, vectors are identified by roman bold type; for example, X , Z(<i>i</i>) . |
| gothic bold | When a name, symbol, or other sequence of characters is used in exactly the same way that it is intended to be entered into the Assembler, a compiler, or other software product, it is printed in a contrasting type style; for example, r20 . |
| <i>gothic italic</i> | Gothic italic type indicates a metasympol that is to be replaced with an item that fulfills the rules for that symbol. |
| [] | Brackets indicate optional arguments or parameters. |
| [] | Brackets, when printed in the contrasting type style, are required, and must be entered as shown. |
| { } | One and only one of the enclosed entries must be selected unless the field is also surrounded by brackets, in which case it is optional. |
| { }... | At least one of the enclosed entries must be selected unless the field is also surrounded by brackets, in which case it is optional. The items may be used in any order, unless otherwise noted. |
| ... | Ellipses indicate that the preceding argument or parameter may be repeated. When an ellipsis follows a right bracket or brace, the entire unit enclosed within the pair of brackets or braces may be repeated. |
| ← | The notation $a \leftarrow b$ means that the value of b is assigned to a . |
| SMALLCAPS | Small capital letters indicate the ASCII name of one of the nondisplayable characters; for example, LF is the linefeed character. |
| ./,\$ | Punctuation other than brackets must be entered as shown. |
| π | $\pi = 3.141592654\dots$ |

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Chapter 1

Introduction

1.1 Descriptions of Primitives

Chapter 2 presents detailed descriptions of each primitive. Each description is divided into sections as follows.

1.1.1 Description of Algorithm

With every primitive is given a brief description of the operation performed. The description uses standard mathematical notation, with the symbol \leftarrow indicating assignment.

1.1.2 Syntax

Primitives are defined in both FORTRAN and C syntax; however, because they adhere to standard calling conventions, all primitives are callable by other high-level languages as well as by the i860 Microprocessor Assembler. Parameters (except complex) are passed by value. Complex parameters are passed by reference.

The keyword `complex` is used as a type in the C examples for complex primitives. Because C does not have `complex` as a primitive type, the following `typedef` is assumed:

```
typedef struct {float re, im} complex;
```

...where **re** is the real part and **im** is the imaginary part¹.

In keeping with this definition of complex variables, when it is necessary to refer to the real and imaginary parts separately, the suffixes `.re` and `.im` are used. For example: `x.re` refers to the real part of the variable `x`, `x.im` refers to the imaginary part of the variable `x`, and `x = (x.re,x.im)`.

Assembly-language programmers can derive register usage from the high-level language syntax given. All parameter passing conforms to the standards specified in *i860™ 64-Bit Microprocessor Programmer's Reference Manual*. Single-precision primitives return values in **f16**; double-precision primitives return values in **f17:f16**; complex primitives leave results in a location specified by an address in **r16**.

1.1.3 Input

The syntax section defines the number and type of input parameters; however, any additional information about the inputs (units and valid range of values, for example) is presented in this section. All angles are expressed in radians.

In order to achieve maximum performance, inputs are not checked for validity. Invalid inputs may produce exceptions, in which case the results depend on the exception handler used.

1.1.4 Output

The syntax section defines the type of the output; however, any additional information about the output (range of possible values, for example) is presented in this section. All angles are expressed in radians.

1. For programming in C, an include file `fmath.h` is provided containing this `typedef` declaration.

1.1.5 Precision

Precision is specified as the number of significant decimal digits.

1.1.6 Timing Estimates

All clock counts shown are approximate timing estimates and are subject to change.

1.2 Primitives by Function

Table 1-1 provides a cross-reference of primitives organized by function. Use this table to find the identifier of the primitive that satisfies the required function and type of operands. Then look up the primitive in Chapter 2, which is arranged in alphabetical order by primitive identifier.

Table 1-1. Primitives by Function

| Function | Type of Arguments | | |
|-------------------------|-------------------|---------|---------|
| | Single | Double | Complex |
| Square Root | sqrt | dsqrt | csqrt |
| Cosine | cos | dcos | ccos |
| Sine | sin | dsin | csin |
| Tangent | tan | dtan | |
| Sine and Cosine | sincos | dsincos | |
| Arc Cosine | acos | dacos | |
| Arc Sin | asin | dasin | |
| Arc Tangent | atan | datan | |
| Arc Tangent of Quotient | atan2 | datan2 | |
| Hyperbolic Cosine | cosh | dcosh | |
| Hyperbolic Sine | sinh | dsinh | |
| Hyperbolic Tangent | tanh | dtanh | |
| Exponential | exp | dexp | cexp |
| Natural Logarithm | alog | dlog | clog |
| Log Base 10 | alog10 | dlog10 | |

1.3 Linking to the Math Library

The name of the library file is **libfm.a**. When programming in the C Language link this library file in the following order:

libfm.a {C Libraries}

When programming in the Fortran Language link it in the following order:

libfm.a {Fortran Libraries}

Chapter 2 Math Primitives

2.1 `acos` Inverse Cosine

Computes...

$$y \leftarrow \text{Arc cos } x$$

... where x and y are single-precision real numbers.

Syntax

| C | FORTRAN |
|--|--|
| <hr/> float x, y ; $y = \text{acosf}(x)$; <hr/> | <hr/> REAL x, y $y = \text{acos}(x)$ <hr/> |

Input $-1 \leq x \leq 1$; zeros and denormals return $\pi/2$; infinities are invalid.

Output $0 \leq y \leq \pi$

Precision 6.52 digits

Timing 63, 111 clocks

Exceptions

| Type | Condition | Masked Response |
|------|------------------|------------------------|
| I | x out of range | QNaN <i>indefinite</i> |

2.2 `alog` Natural Logarithm

Computes...

$$y \leftarrow \log_e x$$

...where x and y are single-precision real numbers.

Syntax

| C | FORTRAN |
|--|---|
| <hr/> float x, y ; $y = \log f(x)$; <hr/> | <hr/> REAL x, y $y = \log(x)$ <hr/> |

Input

$x > 0$.

Output

$-\infty < y < +\infty$

Precision

6.71 digits

Timing

76 clocks

Exceptions

| Type | Condition | Masked Response |
|------|------------------|------------------------|
| I | x out of range | QNaN <i>indefinite</i> |

2.3 `alog10` Log Base 10

Computes...

$$y \leftarrow \log_{10} x$$

...where x and y are single-precision real numbers.

Syntax

C

```
float x, y;  
y = log10f (x);
```

FORTRAN

```
REAL x, y  
y = log10 (x)
```

Input

$x > 0$.

Output

$-\infty < y < +\infty$

Precision

7.07 digits

Timing

76 clocks

Exceptions

| Type | Condition | Masked Response |
|------|------------------|------------------------|
| I | x out of range | QNaN <i>indefinite</i> |

2.4 **asin**Inverse Sine

Computes...

$$y \leftarrow \text{Arc sin } x$$

...where x and y are single-precision real numbers.

Syntax

C

float x, y ;
 $y = \text{asinf}(x)$;

FORTRAN

REAL x, y
 $y = \text{asin}(x)$

Input

$-1 \leq x \leq 1$; zeros and denormals return $\pi/2$; infinities are invalid.

Output

$-\pi/2 \leq y \leq \pi/2$

Precision

6.72 digits

Timing

60, 108 clocks

Exceptions

| Type | Condition | Masked Response |
|------|------------------|------------------------|
| I | x out of range | QNaN <i>indefinite</i> |

2.5 atanInverse Tangent

Computes...

$$y \leftarrow \text{Arc tan } x$$

...where x and y are single-precision real numbers.

Syntax

C

float x, y ;
 $y = \text{atanf}(x)$;

FORTRAN

REAL x, y
 $y = \text{atan}(x)$

Input

$-\infty \leq x \leq +\infty$

Output

$-\pi/2 \leq y \leq \pi/2$; $-\infty$ returns $-\pi/2$

Precision

6.94 digits

Timing

57, 85 clocks

Exceptions

None.

2.6 atan2Inverse Tangent of Quotient

Computes...

$$z \leftarrow \text{Arc tan}(x/y)$$

...where x and y are single-precision real numbers.

Syntax

C

```
float x, y, z;
z = atan2f(x, y);
```

FORTRAN

```
REAL x, y, z
z = atan2(x, y)
```

Input

x and y not both 0.

Output

In the range of π (points just below the negative x -axis) to π (points at or just above the negative x -axis). The range of the result depends on the relationship between the operands according to Table 2-1.

Precision

6.94 digits

Timing

11, 12, 101, 130 clocks

Table 2-1. Results of atan2

| Sign(y) | Sign(x) | $ y < x $? | Final Result |
|---------|---------|---------------|---|
| + | + | Yes | $0 \leq \text{atan}(y/x) \leq \pi/4$ |
| + | + | No | $\pi/4 \leq \text{atan}(y/x) \leq \pi/2$ |
| + | - | No | $\pi/2 \leq \text{atan}(y/x) \leq 3\pi/4$ |
| + | - | Yes | $3\pi/4 < \text{atan}(y/x) < \pi$ |
| - | + | Yes | $-\pi/4 \leq \text{atan}(y/x) \leq 0$ |
| - | + | No | $-\pi/2 \leq \text{atan}(y/x) \leq -\pi/4$ |
| - | - | No | $-3\pi/4 \leq \text{atan}(y/x) \leq -\pi/2$ |
| - | - | Yes | $-\pi \leq \text{atan}(y/x) \leq -3\pi/4$ |

Exceptions

| Type | Condition | Masked Response |
|------|------------------------------------|------------------------|
| I | $x = \pm\infty$ or $y = \pm\infty$ | QNaN <i>indefinite</i> |
| X | $x = y = 0$ | QNaN <i>indefinite</i> |
| I | invalid inputs | QNaN <i>indefinite</i> |

2.7 **ccos** **Complex Cosine**

Computes...

$$y \leftarrow \cos x$$

... where x and y are complex numbers.

$$y.re = \cos(x.re) * \cosh(x.im)$$

$$y.im = -\sin(x.re) * \sinh(x.im)$$

Syntax

C

complex x , y ;
 $y = c_cos(x)$;

FORTRAN

COMPLEX x , y
 $y = ccos(x)$

Input

$-\infty < x.re < +\infty$; $-89.415985 \leq x.im \leq +89.415985$

Output

Precision

7.00 digits for the real part, 7.00 digits for the imaginary part

Timing

231–285 clocks

Exceptions

| Type | Condition | Masked Response |
|------|----------------------|---|
| O | $ x.im > 89.415985$ | $y.re = \pm\infty$, $y.im = \pm\infty$ |
| I | invalid inputs | QNaNindefinite |

2.8 `cexp` Complex Exponential

Computes...

$$y \leftarrow e^x$$

...where x and y are complex numbers.

$$y.re = \exp(x.re) * \cos(x.im)$$

$$y.im = \exp(x.re) * \sin(x.im)$$

Syntax

C

complex x , y ;
`y = c_exp (x)`;

FORTRAN

COMPLEX x , y
`y = CALL cexp (x)`

Input

$-87.336545 < x.re < 88.722839$; $-\infty < x.im < +\infty$

Output

Precision

6.70 digits

Timing

130–150 clocks

Exceptions

| Type | Condition | Masked Response |
|------|---------------------|------------------------|
| O | $x.re > 88.722839$ | $+\infty$ |
| U | $x.re < -87.336544$ | zero |
| I | invalid inputs | QNaN <i>indefinite</i> |

2.9 clog Complex Natural Logarithm

Computes...

$$y \leftarrow \log_e x$$

...where x and y are complex numbers.

$$y.re = 0.5 * \text{alog}(x.re^2 + x.im^2)$$

$$y.im = \text{atan2}(x.im, x.re)$$

Syntax

C

complex x, y ;
 $y = \text{c_log}(x)$;

FORTRAN

COMPLEX x, y
 $y = \text{clog}(x)$

Input

$x.re \neq 0$; $x.im \neq 0$; $x.re$ and $x.im$ not both zero

Output

Precision

7.00 digits for the imaginary part; 6.00 digits for the real part

Timing

187–216 clocks

Exceptions

Trap if $x.re \neq 0$ or $x.im \neq 0$. *[What kind of]* trap if $x.re = x.im = 0$.

| Type | Condition | Masked Response |
|------|----------------|------------------------|
| I | invalid inputs | QNaN <i>indefinite</i> |

2.10 **cos** **Cosine**

Computes...

$$y \leftarrow \cos x$$

...where x and y are single-precision real numbers.

Syntax

C

```
float x, y;
y = cosf (x);
```

FORTRAN

```
REAL x, y
y = COS (X)
```

Input Infinities are invalid.

Output $-1 \leq y \leq 1$

Precision 6.92 digits

Timing 30, 55, 58 clocks

Exceptions Trap on infinity.

| Type | Condition | Masked Response |
|------|----------------|------------------------|
| I | invalid inputs | QNaN <i>indefinite</i> |

2.11 **cosh**Hyperbolic Cosine

Computes...

$$y \leftarrow \cosh x$$

...where x and y are single-precision real numbers.

Syntax

C

```
float x, y;  
y = coshf (x);
```

FORTRAN

```
REAL x, y  
y = cosh (x)
```

Input

$-89.415985 < x < 89.415985$

Output

Precision

6.60 digits

Timing

96 clocks

Exceptions

| Type | Condition | Masked Response |
|------|-----------------------|------------------------|
| O | $ x \geq 89.4159865$ | $+\infty$ |
| I | invalid inputs | QNaN <i>indefinite</i> |

2.12 csin Complex Sine

Computes...

$$y \leftarrow \sin x$$

...where x and y are complex numbers.

$$y.re = \sin(x.re) * \cosh(x.im)$$

$$y.im = \cos(x.re) * \sinh(x.im)$$

Syntax

C

complex x , y ;
 $y = c_sin(x)$;

FORTRAN

COMPLEX x , y
 $y = csin(x)$

Input

$-\infty < x.re < +\infty$; $-89.415985 \leq x.im \leq +89.415985$

Output

Precision

7.00 digits for the real part; 7.00 digits for the imaginary part

Timing

131–285 clocks

Exceptions

| Type | Condition | Masked Response |
|------|----------------------|---|
| O | $ x.im > 89.415985$ | $y.re = \pm\infty$, $y.im = \pm\infty$ |
| I | invalid inputs | QNaN <i>indefinite</i> |

2.13 csqrtComplex Square Root

Computes...

$$y \leftarrow \text{SQRT}(x)$$

...where x and y are complex numbers.

Let...

$$r = \text{sqrt}(x.\text{re}^2 + x.\text{im}^2)$$
$$t = \text{atan2}(x.\text{im}, x.\text{re}) * 0.5$$

Then...

$$y.\text{re} = r * \cos(t)$$
$$y.\text{im} = r * \sin(t)$$

Syntax

C

complex x , y ;
 $y = \text{c_sqrt}(x)$;

FORTRAN

COMPLEX x , y
 $y = \text{csqrt}(x)$

Input

$x.\text{re} \neq 0$; $x.\text{im} \neq 0$; $x.\text{re}$ and $x.\text{im}$ not both zero

Output

Precision

6.89 digits for the imaginary part; 6.97 digits for the real part

Timing

220–269 clocks

Exceptions

| Type | Condition | Masked Response |
|------|------------------|------------------------|
| I | x out of range | QNaN <i>indefinite</i> |

2.14 `dacos`Inverse Cosine

Computes...

$$y \leftarrow \text{Arc cos } x$$

... where x and y are double-precision real numbers.

Syntax

C

```
double x, y;  
y = acos (x);
```

FORTRAN

```
DOUBLE PRECISION x, y  
y = acos (x)
```

| | |
|-------------------|---------------------|
| Input | $-1 \leq x \leq 1$ |
| Output | $0 \leq y \leq \pi$ |
| Precision | 15.54 digits |
| Timing | 122, 197 clocks |
| Exceptions | |

| Type | Condition | Masked Response |
|------|------------------|------------------------|
| I | x out of range | QNaN <i>indefinite</i> |

2.15 `dasin` Inverse Sine

Computes...

$$y \leftarrow \text{Arc sin } x$$

...where x and y are double-precision real numbers.

Syntax

C

```
double x, y;  
y = asin (x);
```

FORTRAN

```
DOUBLE PRECISION x, y  
y = asin (x)
```

Input

$-1 \leq x \leq 1$; zeros and denormals return $\pi/2$; infinities are invalid.

Output

$-\pi/2 \leq y \leq \pi/2$

Precision

15.45 digits

Timing

119, 194 clocks

Exceptions

| Type | Condition | Masked Response |
|------|------------------|------------------------|
| I | x out of range | QNaN <i>indefinite</i> |

2.16 **atan**Inverse Tangent

Computes...

$$y \leftarrow \text{Arc tan } x$$

...where x and y are double-precision real numbers.

Syntax

C

```
double x, y;  
y = atan (x);
```

FORTRAN

```
DOUBLE PRECISION x, y  
y = atan (x)
```

Input

$-\infty < x < +\infty$

Output

$-\pi/2 \leq y \leq \pi/2$; $-\infty$ returns $-\pi/2$; $+\infty$ returns $+\pi/2$.

Precision

15.84 digits

Timing

137, 145 clocks

Exceptions

None.

2.17 `atan2`Inverse Tangent of Quotient

Computes...

$$z \leftarrow \text{Arc tan}(x/y)$$

...where x , y , and z are double-precision real numbers.

Syntax

C

```
double x, y, z;  
z = atan2 (x, y);
```

FORTRAN

```
DOUBLE PRECISION x, y, z  
z = atan2 (x, y)
```

Input

x and y not both 0.

Output

In the range of π (points just below the negative x -axis) to π (points at or just above the negative x -axis). The range of the result depends on the relationship between the operands according to Table 2-1.

Precision

15.35 digits

Timing

11, 12, 190, 198 clocks

Exceptions

| Type | Condition | Masked Response |
|------|------------------------------------|------------------------|
| I | $x = \pm\infty$ or $y = \pm\infty$ | QNaN <i>indefinite</i> |
| X | $x = 0$ and $y = 0$ | QNaN <i>indefinite</i> |
| I | invalid inputs | QNaN <i>indefinite</i> |

2.18 `dcos` Cosine

Computes...

$$y \leftarrow \cos x$$

...where x and y are double-precision real numbers.

Syntax

C

`double x, y;`
`y = cos (x);`

FORTRAN

`DOUBLE PRECISION x, y`
`y = cos (x)`

Input Infinities are invalid.

Output $-1 \leq y \leq 1$

Precision 15.48 digits

Timing 55, 77, 80 clocks

Exceptions Trap on infinity.

| Type | Condition | Masked Response |
|------|----------------|------------------------|
| I | invalid inputs | QNaN <i>indefinite</i> |

2.19 **dcosh**Hyperbolic Cosine

Computes...

$$y \leftarrow \cosh x$$

...where x and y are double-precision real numbers.

Syntax

C

```
double x, y;  
y = cosh (x);
```

FORTRAN

```
DOUBLE PRECISION x, y  
y = cosh (x)
```

Input

$-710.4758600 < x < 710.4758600$

Output

Precision

6.52 digits

Timing

129 clocks

Exceptions

| Type | Condition | Masked Response |
|------|------------------------|-----------------|
| O | $ x \geq 710.4758600$ | $+\infty$ |
| I | invalid inputs | QNaNindefinite |

2.20 dexp Exponential

Computes...

$$y \leftarrow e^x$$

...where x and y are double-precision real numbers.

Syntax

C

```
double x, y;  
y = exp (x);
```

FORTRAN

```
DOUBLE PRECISION x, y  
y = exp (x)
```

Input $-707.7032713 < x < 709.7827128$

Output

Precision 15.44 digits

Timing 111 clocks

Exceptions

| Type | Condition | Masked Response |
|------|--------------------|------------------------|
| O | $x > 709.7827128$ | $+\infty$ |
| U | $x < -707.7032713$ | zero |
| I | invalid inputs | QNaN <i>indefinite</i> |

2.21 **dlog** **Natural Logarithm**

Computes...

$$y \leftarrow \log_e x$$

...where x and y are double-precision real numbers.

Syntax

C

```
double x, y;  
y = log (x);
```

FORTRAN

```
DOUBLE PRECISION x, y  
y = log (x)
```

Input

$x > 0$

Output

Precision

15.65 digits

Timing

123 clocks

Exceptions

| Type | Condition | Masked Response |
|------|------------------|------------------------|
| I | x out of range | QNaN <i>indefinite</i> |

2.22 dlog10 Log Base 10

Computes...

$$y \leftarrow \log_{10} x$$

...where x and y are double-precision real numbers.

Syntax

C

```
double x, y;  
y = log10 (x);
```

FORTRAN

```
DOUBLE PRECISION x, y  
y = log10 (x)
```

Input

$x > 0$

Output

Precision

15.65 digits

Timing

123 clocks

Exceptions

| Type | Condition | Masked Response |
|------|------------------|------------------------|
| I | x out of range | QNaN <i>indefinite</i> |

2.23 dsinSine

Computes...

$$y \leftarrow \sin x$$

...where x and y are double-precision real numbers.

Syntax

C

```
double x, y;  
y = sin (x);
```

FORTRAN

```
DOUBLE PRECISION x, y  
y = sin (x)
```

Input

$-\infty < x < +\infty$; infinities are invalid.

Output

$-1 \leq y \leq 1$

Precision

15.41 digits

Timing

55, 77, 80 clocks

Exceptions

Invalid operation trap on infinity.

| Type | Condition | Masked Response |
|------|----------------|------------------------|
| I | invalid inputs | QNaN <i>indefinite</i> |

2.24 dsincosSine and Cosine

Define...

```
struct pair {double y, double z}
```

Computes...

```
pair.y ← sin x
pair.z ← cos x
```

...where x , y , and z are double-precision real numbers.

Syntax

C

```
double x
pair = sincos (x);
```

FORTRAN

Not Applicable

Input $-\infty < x < +\infty$; infinities are invalid.

Output $-1 \leq y \leq 1$

Precision 15.41, 15.48 digits respectively

Timing 106, 128 clocks

Exceptions Invalid operation trap on infinity.

| Type | Condition | Masked Response |
|------|----------------|------------------------|
| I | invalid inputs | QNaN <i>indefinite</i> |

2.25 dsinh.....Hyperbolic Sine

Computes...

$$y \leftarrow \sinh x$$

...where x and y are double-precision real numbers.

Syntax

C

```
double x, y;  
y = sinh (x);
```

FORTRAN

```
DOUBLE PRECISION x, y  
y = sinh (x)
```

Input

In the range $-710.4758600 < x < 710.4758600$

Output

Precision

15.27 digits

Timing

57, 130 clocks

Exceptions

Overflow trap if x out of range, which returns positive infinity{?}.

| Type | Condition | Masked Response |
|------|-----------------------|------------------------|
| O | $x \geq 710.4758600$ | $+\infty$ |
| O | $x \geq -710.4758600$ | $-\infty$ |
| I | invalid inputs | QNaN <i>indefinite</i> |

2.26 dsqrt Square Root

Computes...

$$y \leftarrow \text{SQRT}(x)$$

...where x and y are double-precision real numbers.

Syntax

C

```
double x, y;  
y = sqrt(x);
```

FORTRAN

```
DOUBLE PRECISION x, y  
y = sqrt(x)
```

Input

$$x \geq 0$$

Output

$$y \geq 0; \text{dsqrt}(-0) = -0; \text{dsqrt}(+\infty) = +\infty$$

Precision

15.98 digits

Timing

60 clocks

Exceptions

| Type | Condition | Masked Response |
|------|------------------|------------------------|
| I | x out of range | QNaN <i>indefinite</i> |

2.25 dsinh.....Hyperbolic Sine

Computes...

$$y \leftarrow \sinh x$$

...where x and y are double-precision real numbers.

Syntax

C

double x, y ;
 $y = \sinh(x)$;

FORTRAN

DOUBLE PRECISION x, y
 $y = \sinh(x)$

Input

In the range $-710.4758600 < x < 710.4758600$

Output

Precision

15.27 digits

Timing

57, 130 clocks

Exceptions

Overflow trap if x out of range, which returns positive infinity{?}.

| Type | Condition | Masked Response |
|------|-----------------------|------------------------|
| O | $x \geq 710.4758600$ | $+\infty$ |
| O | $x \geq -710.4758600$ | $-\infty$ |
| I | invalid inputs | QNaN <i>indefinite</i> |

2.26 dsqrt Square Root

Computes...

$$y \leftarrow \text{SQRT}(x)$$

...where x and y are double-precision real numbers.

Syntax

C

```
double x, y;  
y = sqrt(x);
```

FORTRAN

```
DOUBLE PRECISION x, y  
y = sqrt(x)
```

Input $x \geq 0$

Output $y \geq 0$; dsqrt(-0) = -0; dsqrt(+∞) = +∞

Precision 15.98 digits

Timing 60 clocks

Exceptions

| Type | Condition | Masked Response |
|------|------------------|------------------------|
| I | x out of range | QNaN <i>indefinite</i> |

2.27 dtan Tangent

Computes...

$$y \leftarrow \tan x$$

...where x and y are double-precision real numbers.

Syntax

C

```
double x, y;  
y = tan (x);
```

FORTRAN

```
DOUBLE PRECISION x, y  
y = tan (x)
```

Input $-\infty < x < +\infty$, in radians

Output

Precision 15.35 digits

Timing 115, 127, 163, 175 clocks

Exceptions Invalid operation trap on infinity.

| Type | Condition | Masked Response |
|------|----------------|------------------------|
| I | invalid inputs | QNaN <i>indefinite</i> |

2.28 dtanhHyperbolic Tangent

Computes...

$$y \leftarrow \tanh x$$

...where x and y are double-precision real numbers.

Syntax

C

```
double x, y;  
y = tanh (x);
```

FORTRAN

```
DOUBLE PRECISION x, y  
y = tanh (x)
```

Input

Output

Precision 15.35 digits

Timing 20, 59, 128 clocks

Exceptions None.

2.29 exp Exponential

Computes...

$$y \leftarrow e^x$$

... where x and y are single-precision real numbers.

Syntax

C

```
float x, y;  
y = expf (x);
```

FORTRAN

```
REAL x, y  
y = exp (x)
```

Input

$-87.336545 < x < 88.722839$

Output

Precision

6.72 digits

Timing

63 clocks

Exceptions

| Type | Condition | Masked Response |
|------|------------------|------------------------|
| O | $x > 88.722839$ | $+\infty$ |
| U | $x < -87.336544$ | zero |
| I | invalid inputs | QNaN <i>indefinite</i> |

2.30 **sin** **Sine**

Computes...

$$y \leftarrow \sin x$$

... where x and y are single-precision real numbers.

Syntax

C

float x, y ;
 $y = \sin f(x)$;

FORTRAN

REAL x, y
 $y = \sin(x)$

Input

$-\infty < x < +\infty$; infinities are invalid.

Output

$-1 \leq y \leq 1$

Precision

6.92 digits

Timing

30, 55, 58 clocks

Exceptions

Invalid operation trap on infinity.

| Type | Condition | Masked Response |
|------|----------------|------------------------|
| I | invalid inputs | QNaN <i>indefinite</i> |

2.31 **sincos** Sine and Cosine

Define...

```
struct pair { float y, float z }
```

Computes...

```
pair.y ← sin x  
pair.z ← cos x
```

...where x , y , and z are single-precision real numbers.

Syntax

C

```
float x;  
pair = sincosf (x);
```

FORTRAN

Not Applicable

Input

$-\infty < x < +\infty$; infinities are invalid.

Output

$-1 \leq y \leq 1$

Precision

6.92 digits

Timing

54, 74 clocks

Exceptions

Invalid operation trap on infinity.

| Type | Condition | Masked Response |
|------|----------------|------------------------|
| I | invalid inputs | QNaN <i>indefinite</i> |

2.32 `sinh`Hyperbolic Sine

Computes...

$$y \leftarrow \sinh x$$

...where x and y are single-precision real numbers.

Syntax

C

```
float x, y;  
y = sinhf (x);
```

FORTRAN

```
REAL x, y  
y = sinh (x)
```

Input In the range $-89.41598629 < x < 89.41598629$

Output

Precision 6.92 digits

Timing 67, 101 clocks

Exceptions Overflow trap if x out of range, which returns positive infinity.

| Type | Condition | Masked Response |
|------|-----------------------|------------------------|
| O | $x \geq 89.41598629$ | $+\infty$ |
| O | $x \leq -89.41598629$ | $-\infty$ |
| I | invalid inputs | QNaN <i>indefinite</i> |

2.33 `sqrt`Square Root

Computes...

$$y \leftarrow \text{SQRT}(x)$$

...where x and y are single-precision real numbers.

Syntax

C

float x, y ;
 $y = \text{sqrtf}(x)$;

FORTRAN

REAL x, y
 $y = \text{sqrt}(x)$

Input

$x \geq 0$

Output

$y \geq 0$; $\text{sqrt}(-0) = -0$; $\text{sqrt}(+\infty) = +\infty$

Precision

7.31 digits

Timing

37 clocks

Exceptions

| Type | Condition | Masked Response |
|------|------------------|------------------------|
| I | x out of range | QNaN <i>indefinite</i> |

2.34 **tan** **Tangent**

Computes...

$$y \leftarrow \tan x$$

...where x and y are single-precision real numbers.

Syntax

| C | FORTRAN |
|--|---|
| <hr/> float x, y ; $y = \tan f(x)$; <hr/> | <hr/> REAL x, y $y = \tan(x)$ <hr/> |

Input $-\infty < x < +\infty$, in radians

Output $-\pi/2 \leq y \leq +\pi/2$

Precision 6.53 digits

Timing 69, 81, 90, 102 clocks

Exceptions Invalid operation trap on infinity.

| Type | Condition | Masked Response |
|-------------|------------------|------------------------|
| I | invalid inputs | QNaN <i>indefinite</i> |

2.35 **tanh**Hyperbolic Tangent

Computes...

$$y \leftarrow \tanh x$$

...where x and y are single-precision real numbers.

Syntax

C

```
float x, y;  
y = tanhf (x);
```

FORTRAN

```
REAL x, y  
y = tanh (x)
```

Input

$-\infty \leq x \leq +\infty$

Output

$-1 \leq y \leq +1$

Precision

6.92 digits

Timing

20, 29, 103 clocks

Exceptions

None.

